Observing Extra-Solar Planets

A. When a planet orbits a star, it causes the star to "wobble". That is because the planet and star both exert a gravitational force on each other. The star's force on the planet causes it to move in an orbit around the star; the planet's force on the star causes it to move in a much smaller orbit around the center of mass of the system. This small motion shows up as a "wobble" in the star.

B. As one observes the star, sometimes it is moving toward you, sometimes away from you. This motion can be detected as a doppler shift in the spectral lines of the star.

C. The doppler equation: \( \frac{\Delta \lambda}{\lambda_o} = \frac{v}{c} \), where \( \Delta \lambda \) is the shift of the wavelength from the original wavelength (\( \lambda_o \)), and \( v \) is the speed of the star, \( c \) is the speed of light, gives the speed of the star. (With one correction, to be described later.)

D. From the velocity-time graph, the period of the orbit can be found from length of time between peaks. Knowing the period of the orbit, Kepler's third law: \( r^3 = \frac{GM_{\text{star}}}{4\pi^2} P^2 \) gives the radius of the planet's orbit. The mass of the star can be determined by its spectral type.

E. Now, gravity is the force that is pulling the planet into an orbit, so gravity is providing a centripetal force. So \( \frac{m_p v_p^2}{r} = \frac{GM_{\text{star}} m_p}{r^2} \). This gives: \( v_p = \sqrt{\frac{GM_{\text{star}}}{r}} \).

F. As the star pulls the planet one way, the planet pulls the star the other. This is Newton's third law, and it also gives rise to the conservation of momentum, since only internal forces act. So the momentum the planet gains in one way equals the momentum the star gains in the other way: \( m_p v_p = M_{\text{star}} v_{\text{star}} \). From this we find the mass of the planet: \( m_p = \frac{M_{\text{star}} v_{\text{star}}}{v_p} \).

G. The correction to this is that only the radial velocity of the star is measured by the doppler effect. If the planet does not orbit the star in an orbit seen exactly edge-on, then the velocity measured will be too small, by a factor of \( \sin(i) \). What is measured is actually called \( K \), the component of the speed parallel to the line of sight. So \( K = v_{\text{star}} = v_{\text{true}} \sin(i) \), and the mass measured is actually \( m_p \sin(i) \). Unfortunately, \( i \) cannot be found, generally.
Extra Solar Planets Worksheet

Name __________________________

The figure below shows actual data of the "wobble" velocity of the star 47 UMa due to the motion of a planet around the star, taken by astronomers at the Lick Observatory in 1988-2000. Using this data, estimate the mass of the planet.

- Measure the period of the planet (the time to make one orbit) in years, and convert to seconds.

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  \text{Period} = \text{______________ years} = \text{______________ seconds}
  \]

- The star 47 UMa is a main sequence star similar to the sun, with a spectral type of G0V. Using this information (and Table 13.3 and Appendix 2 in your text) and the period you just found, use the equation \( r^3 = \frac{GM_{\text{star}} P^2}{4 \pi^2} \) to find the radius of the planet's orbit.

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  \text{Star's Mass} = \text{______________ kg} \quad \text{r} = \text{______________ m}
  \]

- Calculate the speed of the planet around the star using the equation: \( v_p = \sqrt{\frac{GM_{\text{star}}}{r}} \).

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  v_p = \text{______________ m/s}
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- Calculate the mass of the planet using \( m_p = \frac{M_{\text{star}} v_{\text{star}}}{v_p} \), where \( v_{\text{star}} \) is the maximum speed of the star, as found on the graph. Convert this mass to Jupiter masses, where the mass of Jupiter is \( 1.9 \times 10^{27} \) kg.

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  m_{\text{planet}} = \text{______________ kg} = \text{______________ Jupiter masses}
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