Show all work!

Remember to include vectors while working the problem, and in solutions!
If you use symmetry to simplify or eliminate integrals, be sure to carefully and explicitly justify the symmetry arguments!

1. A thin rod of length $L$ lies on the $+x$ axis, with its closest end a distance $a$ from the origin. The rod carries a linear charge density $\lambda$, and a total charge $Q$. Find expressions for the electric field at the origin, in terms of $\lambda$ and in terms of $Q$.

![Diagram of a rod on the x-axis]({}#)

2. A uniform shell of total charge $-Q$ surrounds a uniform sphere of total charge $+Q$. The sphere has a radius of $a$, while the shell has inner and outer radii of $2a$ and $4a$ as shown. Find the electric field $E$ in the middle of the shell, at a distance $3a$ from the center.

![Diagram of a shell around a sphere]({}#)

3. If at the Earth’s surface the electric field is 100 N/C pointed downward, and the magnetic field is $5.0 \times 10^{-5}$ T pointed northward, at what minimum velocity (size and direction) must a proton travel to experience no Lorentz force? (Ignore gravity and air resistance.)
4. A cylindrical rod aligned with the +z-axis carries a current density \( \vec{J} = J_0 t^{1/2} \hat{z} \), where \( J_0 \) is a constant and \( r \) is the distance from the central axis of the cylinder. Find the field \( \vec{B} \):

a) inside the cylinder, and

b) outside the cylinder in terms of \( J_0 \).

Extra credit: If the total current in the wire is \( I \), write the field inside and outside the cylinder in terms of \( I \).

5. a) Explain the physical meaning of the divergence \( \nabla \cdot \vec{A} \) of a field. Explain the physical meaning of the curl \( \nabla \times \vec{A} \). (Assume for this part that \( \vec{A} \) is an arbitrary field, not specifically an electric or magnetic field.)

b) Use your answers to (a) to explain in words the physical meaning of the differential form of three equations: (1) Gauss’ Law for Electric Fields, (2) Gauss’ Law for Magnetic Fields, and (3) Ampere’s Law.

Some constants…

\[
\begin{align*}
c &= 3.0 \times 10^8 \text{ m/s} & h &= 6.626 \times 10^{-34} \text{ Js} & e &= 1.6 \times 10^{-19} \text{ C} \\
\mu_0 &= 4\pi \times 10^{-7} \text{ Nm}^2/\text{C}^2 & \varepsilon_0 &= 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2 & m_{\text{electron}} &= 9.1 \times 10^{-31} \text{ kg} \\
m_{\text{proton}} &= 1.67 \times 10^{-27} \text{ kg} & m_{\text{earth}} &= 6.0 \times 10^{24} \text{ kg}
\end{align*}
\]