The differential equations describing the mass, pressure and luminosity in a star come from the basic definitions and the basic physics. The relations are:

\[
\frac{dP}{dr} = -\frac{Gm(r)\rho(r)}{r^2} \quad \text{(The equation of hydrostatic equilibrium)}
\]

\[
\frac{dm}{dr} = 4\pi r^2 \rho(r) \quad \text{(The definition of density)}
\]

\[
\frac{dL}{dr} = 4\pi r^2 \varepsilon(r) \quad \text{(The definition of Luminosity)}
\]

In addition, if we assume that the star is composed of an ideal gas (a good approximation for solar-type stars) then we have:

\[
PV = n k T, \quad \text{or} \quad T = \frac{P V}{k n}.
\]

Now, \( n \) is the number of particles in the star, so \( n/V \) is the number density:

\[
\frac{n}{V} = \frac{\rho}{\overline{m}}, \quad \text{where} \quad \overline{m} \text{ is the average mass of particles in the star. We can assume for the sun that} \quad \overline{m} = 1.02 \times 10^{-27} \text{ kg.}
\]

So

\[
T(r) = \frac{P(r)\overline{m}}{k \rho(r)}
\]

Finally, the luminosity profile can be calculated if the power density profile \( \varepsilon(r) \) is known. For hydrogen burning in the proton-proton chain,

\[
\varepsilon(r) = 1.098 \times 10^{-37} \rho(r)^2 T(r)^4 \text{ Watts} / \text{m}^3.
\]

So in practice, one develops a model for the density profile \( \rho(r) \). Then the mass profile is calculated by integrating the second equation:

\[
m(r) = \int_0^r 4\pi r^2 \rho(r) dr.
\]

This gives the amount of mass inside of radius \( r \). So \( m(0) = 0 \).
Next, one can find the pressure profile by integrating the equation of hydrostatic equilibrium:

\[ P(r) = \int_0^r \frac{Gm(r)\rho(r)}{r^2} \, dr \]

This finds the pressure at radius \( r \). The central pressure must be estimated from other sources, or found by manipulating the model to give a reasonable figure. The standard Solar Model gives a central pressure of \( P(0) = 1.65 \times 10^{16} \) Pa. You should start with this value, but add a constant to it so that the outside pressure \( P(R) = 0 \). This adjustment gives an estimate for the central pressure, and a way to compare your model with others.

The temperature profile is found directly from the equation of state:

\[ T(r) = \frac{P(r)\bar{m}}{k\rho(r)} \]

There are no free parameters here, so this gives another good comparison between your model and others. Notice that we are assuming a uniform, isotropic model for the sun that does not take into account opacity changes at the photosphere, so your model will probably not give a correct value for the surface temperature of the sun.

Finally the luminosity can be found by integrating the luminosity equation and using the expression for the power density from hydrogen fusion via the proton-proton chain:

\[ L(r) = \int_0^r 4\pi r^2 \varepsilon(r) \, dr \quad \text{where} \quad \varepsilon(r) = 1.098 \times 10^{-37} \rho(r)^2 T(r)^4 \text{Watts/m}^3. \]

The luminosity is the amount of power generated per square meter, so \( L(0) = 0 \).

With the proper values of three constants: \( k = 1.381 \times 10^{-23} \) J/K, \( G = 6.67 \times 10^{-11} \) Nm\(^2\)/kg\(^2\) and \( R = 7.0 \times 10^8 \) m (radius of the sun) the models can be found in SI units.

Your assignment is to take one density profile and calculate the mass, pressure, temperature and luminosity profiles. You should use about 20 linearly spaced steps in radius (ie 0, 0.05R, 0.1R, etc). First calculate the mass profile. Then calculate the pressure profile, and adjust the central pressure to achieve nearly zero pressure at the outside of the star. Then calculate the temperature and luminosity profiles. Make graphs of each profile. Finally, write a short description of what you did, of the value for the central pressure you found, and of how your model compared with the solar models in chapter 5 of your text.